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2008 J. Phys. A: Math. Theor. 41 072002

(http://iopscience.iop.org/1751-8121/41/7/072002)

IOP FTC
IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 072002 (9pp) doi:10.1088/1751-8113/41/7/072002

FAST TRACK COMMUNICATION

Decoherence bypass of macroscopic superpositions in quantum measurement

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Received 26 November 2007, in final form 16 December 2007
Published 5 February 2008
Online at stacks.iop.org/JPhysA/41/072002

Abstract
We study a class of quantum measurement models. A microscopic object is entangled with a macroscopic pointer such that a distinct pointer position is tied to each eigenvalue of the measured object observable. Those different pointer positions mutually decohere under the influence of an environment. Overcoming limitations of previous approaches we (i) cope with initial correlations between pointer and environment by considering them initially in a metastable local thermal equilibrium, (ii) allow for object–pointer entanglement and environment-induced decoherence of distinct pointer readouts to proceed simultaneously, such that mixtures of macroscopically distinct object–pointer product states arise without intervening macroscopic superpositions, and (iii) go beyond the Markovian treatment of decoherence.

PACS numbers: 03.65.Ta, 03.65.Yz

1. Introduction

The interpretation and theoretical description of measurements on quantum systems have been under debate since the birth of quantum theory [1]. More recently, interest in this question has been revived by new developments in quantum information. Quantum detection can be used either to extract information on quantum states or to monitor quantum systems (quantum trajectories [2], quantum Zeno effect [3]). Quantitative treatments of measurement models serve both to elucidate the self-consistency of quantum theory and its interpretation, and to calculate the time scales relevant for experiments. Data for the decoherence time are in fact accumulating, in microwave cavities [4], in solid-state devices like superconducting tunnel junction nanocircuits [5, 6], and in electron beams interacting with a semiconducting plate [7]. On a more fundamental ground, ever larger classes of nonlocal hidden-variable theories are being ruled out experimentally as competitors of quantum mechanics [8].

We propose, in this communication, to extend the range of validity of the approach by Zeh, Zurek, and others based on environment-induced decoherence [9–14]. In this vein, i.e., on the basis of the probabilistic interpretation of quantum mechanics (Born rules) and of a lack of knowledge about the microscopic degrees of freedom of the apparatus, we demonstrate that the reading of a macroscopic pointer of the apparatus reveals an eigenvalue of an observable of the measured quantum object, in spite of the unitary evolution of the composite system (object and apparatus). We obtain explicit results for the object–pointer dynamics for a class of models under realistic assumptions. We would like to note that macroscopic manifestations of microscopic quantum fluctuations are not the exclusive privilege of measurement: they also appear e.g. in superfluorescence, where intense light pulses display substantial shot-to-shot fluctuations [15] and in cosmic rays, where single particles propagating in a fluctuating spacetime generate large showers of particles in the atmosphere.

During an ideal measurement the quantum object ($\mathcal{S}$) interacts with the pointer ($\mathcal{P}$) in such a way that a one-to-one correspondence arises between the eigenvalues of the measured object observable and macroscopically distinct pointer states. The coupling of $\mathcal{P}$ with its environment ('bath' $\mathcal{B}$) causes decoherence. Most previous work deals with these two interactions separately (see, however, [13, 14]): a first step ('premeasurement') exclusively treats the unitary evolution entangling $\mathcal{S}$ and $\mathcal{P}$ and yields a superposition of the macroscopically distinct object–pointer states associated with each eigenvalue of the measured observable. That latter 'Schrödinger cat' state is taken as the initial state for a second process, decoherence; there, the quantum correlations between object and apparatus are transformed into classical correlations, as the superposition of object–pointer states is degraded to a statistical mixture of the same states. For such a sequential treatment to make physical sense, the time duration of the entanglement process would have to be short compared with the decoherence time $t_{\rm dec}$. But since the latter tends to be very small for macroscopic superpositions, that assumption is quite questionable. A second shortcoming of some previous work lies in the assumption of initial statistical independence of $\mathcal{P}$ and $\mathcal{B}$; since these two systems cannot be isolated from each other, a more realistic assumption is a metastable local thermal state of $\mathcal{P}+\mathcal{B}$. As a third restriction, memory effects are often neglected for the (reduced) object–pointer dynamics. That Markov approximation assumes $t_{\rm dec}$ to be larger than the bath correlation time $T_{\mathcal{B}}$, a condition not satisfied in some experiments [5, 6] and of questionable validity if macroscopic or even mesoscopic superpositions or mixtures arise during the object–pointer evolution.

We overcome the three aforementioned deficiencies. The key is an assumption for a certain ordering of time scales: the decoherence time $t_{\rm dec}$ and the object–pointer interaction time $t_{\rm int}$ must be small compared to the characteristic time $T_{\mathcal{S}}$ ($T_{\mathcal{P}}$) of the evolution of the measured observable S (the pointer position X) under the object Hamiltonian $H_{\mathcal{S}}$ (the pointer Hamiltonian $H_{\mathcal{P}}$),

$$t_{\rm dec}, t_{\rm int} \ll T_{\mathcal{S}}, \qquad t_{\rm dec}, t_{\rm int}, \hbar\beta \ll T_{\mathcal{P}}. \tag{1}$$

In the last limit, $\beta = (k_B T)^{-1}$ denotes the inverse temperature of the bath. As desirable for a measurement, the free dynamics of $\mathcal{S}$ then remains ineffective on S during the measurement, i.e., $S^0(t) = \mathrm{e}^{\mathrm{i}H_{\mathcal{S}}t/\hbar} S\, \mathrm{e}^{-\mathrm{i}H_{\mathcal{S}}t/\hbar} \simeq S$ for $t \lesssim t_{\rm dec}, t_{\rm int}$. Here $S^0(t) \simeq S$ means that the expectation value of $S^0(t_1)\cdots S^0(t_n)$ in the object initial state is approximately equal to the nth moment of S, for any $0 \leqslant t_1, \ldots, t_n \lesssim t_{\rm dec}, t_{\rm int}$. Similarly, the free dynamics of $\mathcal{P}$ is ineffective to move the pointer between $t = 0$ and $t \approx t_{\rm dec}, t_{\rm int}$. Note that $T_{\mathcal{S}} = \infty$ if S commutes with $H_{\mathcal{S}}$. The conditions on $T_{\mathcal{S}}$ in (1) are necessary in order that an eigenstate $|s\rangle$ of S be left almost unchanged by the measurement. The conditions on $T_{\mathcal{P}}$ (in particular, the high-temperature limit $\hbar\beta \ll T_{\mathcal{P}}$) would be difficult to avoid for a macroscopic pointer.

A further key input is the quantum central limit theorem (QCLT) [16] which implies Gaussian statistics (Wick theorem) for the bath coupling agent in the pointer–bath interaction, as discussed below. The limits (1) and the QCLT imply validity of our results for a wide class of objects and apparatus. An extreme case of universality [17] arises under the further stipulation that entanglement and decoherence be faster than the decay of bath correlations (time constant $T_{\mathcal{B}}$). Indeed, for $t_{\rm dec}, t_{\rm int} \ll T_{\mathcal{B}}$ even the free bath motion remains ineffective during the measurement. Opposite to that slow-bath limit is the Markov regime $T_{\mathcal{B}} \ll t_{\rm dec}, t_{\rm int}$. Our analysis below covers both these regimes as well as the intermediate case $t_{\rm dec}, t_{\rm int} \approx T_{\mathcal{B}}$.

2. The model

2.1. Object, pointer and bath

We consider a three-partite system comprising the microscopic object $\mathcal{S}$, a single-degree-of-freedom macroscopic pointer $\mathcal{P}$ and a bath $\mathcal{B}$ with many ($N \gg 1$) degrees of freedom (labelled by ν). The following dynamical variables will come into play: for $\mathcal{S}$, the observable S to be measured, assumed to have a discrete spectrum; for $\mathcal{P}$, the position X and momentum P; and for $\mathcal{B}$, a certain coupling agent B. The pointer is coupled to $\mathcal{S}$ and $\mathcal{B}$ by the Hamiltonians

$$H_{\mathcal{PS}} = \epsilon S P, \qquad H_{\mathcal{PB}} = B X, \qquad B = N^{-1/2} \sum_{\nu=1}^{N} B_\nu. \tag{2}$$

The object–pointer coupling Hamiltonian $H_{\mathcal{PS}}$ is chosen so as to (i) not change the measured observable S (i.e., $[H_{\mathcal{PS}}, S] = 0$); (ii) be capable of shifting the pointer position by an amount proportional to S, such that each eigenvalue s of S becomes tied up with a specific pointer reading; (iii) involve a large coupling constant ϵ, so that different eigenvalues $s \neq s'$ end up associated with pointer readings separated by large distances. The pointer–bath interaction is chosen for most efficient decoherence of distinct pointer positions [17]; the additivity of B in contributions B_ν acting on single degrees of freedom of the bath will allow us to invoke the QCLT.

Let us point out an essential difference between our model and the interacting spin model of [14]. Unlike in this reference, $\mathcal{S}$ is strongly coupled to a single degree of freedom (the pointer $\mathcal{P}$) of the apparatus, e.g. with its total momentum P in a given direction. The coupling of $\mathcal{S}$ with the other apparatus degrees of freedom (the bath $\mathcal{B}$, for us) is assumed to be much weaker and can therefore be neglected (see [18]).

The full Hamiltonian is $H = H_{\mathcal{S}} + H_{\mathcal{P}} + H_{\mathcal{B}} + H_{\mathcal{PS}} + H_{\mathcal{PB}}$. We need not specify $H_{\mathcal{S}}$. The pointer Hamiltonian $H_{\mathcal{P}} = P^2/(2M) + V(X)$ must allow for a well-defined rest state. We assume that $V(x)$ has a local minimum at $x = 0$, i.e., $V'(0) = 0$ and $V''(0) > 0$. The time scale for free pointer motion then is the period $T_{\mathcal{P}} = 2\pi(M/V''(0))^{1/2}$ of oscillations around this minimum. Like the coupling agent B, the bath Hamiltonian $H_{\mathcal{B}}$ is assumed additive as $H_{\mathcal{B}} = \sum_\nu H_{\mathcal{B},\nu}$ with $[H_{\mathcal{B},\nu}, B_\mu] \propto \delta_{\mu\nu}\dot{B}_\mu$ and $[H_{\mathcal{B},\mu}, H_{\mathcal{B},\nu}] = 0$, again clearing ground for the QCLT.

2.2. Initial state: pointer localized around $x = 0$, apparatus in thermal equilibrium

It is appropriate to require initial statistical independence of the object and the apparatus, and thermal equilibrium for the apparatus. The initial density operator $\rho_{\mathcal{S}}(0)$ of the object may represent a pure or a mixed state. The full initial density operator reads

$$\rho(0) = \rho_{\mathcal{S}}(0) \otimes \rho_{\mathcal{PB}}(0), \qquad \rho_{\mathcal{PB}}(0) = Z_{\mathcal{PB}}^{-1}\, \mathrm{e}^{-\beta(H_{\mathcal{P}}+H_{\mathcal{B}}+H_{\mathcal{PB}})}. \tag{3}$$

By invoking the high-temperature limit $\hbar\beta \ll T_{\mathcal{P}}$ and the Gaussian statistics of B (as implied by the QCLT) and by tracing out the bath we find (see below) the reduced density matrix $\rho_{\mathcal{P}}(0) = \mathrm{tr}_{\mathcal{B}}\,\rho_{\mathcal{P}\mathcal{B}}(0)$ of the pointer in the position representation,

$$\langle x|\rho_{\mathcal{P}}(0)|x'\rangle \propto \mathrm{e}^{-\beta(V_{\mathrm{eff}}(x)+V_{\mathrm{eff}}(x'))/2}\,\mathrm{e}^{-2\pi^2(x-x')^2/\lambda_{\mathrm{th}}^2} \tag{4}$$

where $\lambda_{\mathrm{th}} = 2\pi\hbar(\beta/M)^{1/2}$ is the thermal de Broglie wavelength. The pointer potential here appears renormalized by the pointer–bath interaction as

$$V_{\mathrm{eff}}(x) = V(x) - (\gamma_0/\hbar)x^2, \qquad \gamma_0 = \int_{-\infty}^{0} \mathrm{d}t\,\Im h(t) \geqslant 0 \tag{5}$$

where $h(t)$ is the autocorrelator of the bath coupling agent with respect to the free bath thermal state $\rho_{\mathcal{B}}^{(\mathrm{eq})} \propto \mathrm{e}^{-\beta H_{\mathcal{B}}}$,

$$h(t) = \langle \tilde{B}(t)B\rangle_0 = \mathrm{tr}_{\mathcal{B}}\,\tilde{B}(t)B\rho_{\mathcal{B}}^{(\mathrm{eq})}, \qquad \tilde{B}(t) = \mathrm{e}^{\mathrm{i}H_{\mathcal{B}}t/\hbar}B\,\mathrm{e}^{-\mathrm{i}H_{\mathcal{B}}t/\hbar} \tag{6}$$

and we assume $\langle B\rangle_0 = 0$. For stability of the whole apparatus the pointer–bath coupling must be weak enough so that $V''_{\mathrm{eff}}(0) > 0$; we even bound the latter curvature finitely away from zero by, say, $V''_{\mathrm{eff}}(0) > V''(0)/2$, i.e.,

$$\gamma_0/\hbar < V''(0)/4. \tag{7}$$

This makes sure that the initial density $\langle x|\rho_{\mathcal{P}}(0)|x\rangle$ of pointer positions has a single peak at $x = 0$ with a renormalized width $\Delta_{\mathrm{eff}} = [\beta(V''(0) - 2\gamma_0/\hbar)]^{-1/2}$ of the order of the bare thermal fluctuation $\Delta_{\mathrm{th}} = (\beta V''(0))^{-1/2}$.

If $V(x) = \mathrm{o}(x^2)$ at large distances $|x|$, the effective potential $V_{\mathrm{eff}}(x)$ is unstable. The matrix elements (4) then correspond to (the reduced pointer state of) a *local thermal equilibrium*. That local equilibrium for the apparatus can be achieved by preparing $\mathcal{P}$ in some state localized near $x = 0$ at time $t = -t_{\mathrm{i}}$ and then letting it interact with $\mathcal{B}$ between $t = -t_{\mathrm{i}}$ and $t = 0$. If the thermalization time is small compared with the tunnelling escape time, one may choose t_{i} larger than the former but much smaller than the latter time, so that $\mathcal{P}$ is still within the effective potential well when the measurement starts at $t = 0$. In order to be able to prepare the apparatus in such a local equilibrium, the height V_0^{eff} of the two potential barriers surrounding the local minimum of $V_{\mathrm{eff}}(x)$ at $x = 0$ must be large compared with the thermal energy β^{-1}. Thanks to (7), this is the case provided that the bare potential $V(x)$ satisfies the same requirement, i.e., $V_0 \gg \beta^{-1}$. Interestingly, $V(x)$ can be chosen such that the two potential barriers of $V_{\mathrm{eff}}(x)$ are separated by a mesoscopic distance $W_{\mathrm{eff}} \approx \left(V_0^{\mathrm{eff}}/V''_{\mathrm{eff}}(0)\right)^{1/2} \gg \Delta_{\mathrm{eff}}$ (so that $V_0^{\mathrm{eff}} \gg \beta^{-1}$) which is small compared with the macroscopic read-out scale Δ_{class}. The object–pointer interaction then just has to get the pointer out of the well, leaving the subsequent displacement growth to the action of the effective pointer potential. The instability resulting from the pointer–bath coupling (2) hence provides an amplification mechanism. For a macroscopic pointer at high temperature ($\hbar\beta \ll T_{\mathcal{P}}$), the different length scales are ordered as $\lambda_{\mathrm{th}} \ll \Delta_{\mathrm{th}} \approx \Delta_{\mathrm{eff}} \ll W_{\mathrm{eff}} \ll \Delta_{\mathrm{class}}$.

3. Object–pointer dynamics

Assuming that the bath state is not ascertainable, we define the object–pointer density matrix at time t as $\rho_{\mathcal{P}\mathcal{S}}(t) = \mathrm{tr}_{\mathcal{B}}\,\mathrm{e}^{-\mathrm{i}Ht/\hbar}\rho(0)\,\mathrm{e}^{\mathrm{i}Ht/\hbar}$. Accepting a relative error $\mathrm{O}(t/T_{\mathcal{P}}, t/T_{\mathcal{S}})$, we simplify the time evolution operator at time $t \ll T_{\mathcal{S}}, T_{\mathcal{P}}$ as $\mathrm{e}^{-\mathrm{i}Ht/\hbar} \simeq U(t)\,\mathrm{e}^{-\mathrm{i}(H_{\mathcal{S}}+H_{\mathcal{P}})t/\hbar}$, with

$$U(t) = \mathrm{e}^{-\mathrm{i}(H_{\mathcal{B}}+H_{\mathcal{P}\mathcal{S}}+H_{\mathcal{P}\mathcal{B}})t/\hbar} = \mathrm{e}^{-\mathrm{i}H_{\mathcal{B}}t/\hbar}\,\mathrm{e}^{-\mathrm{i}\epsilon SPt/\hbar}\left(\mathrm{e}^{-\mathrm{i}\int_0^t \mathrm{d}\tau\,(X+\epsilon S\tau)\tilde{B}(\tau)/\hbar}\right)_+. \tag{8}$$

Here, $(\cdot)_+$ denotes time ordering; the momentum P was used as the generator of pointer displacements, $\mathrm{e}^{\mathrm{i}\epsilon SPt/\hbar}X\mathrm{e}^{-\mathrm{i}\epsilon SPt/\hbar} = X + \epsilon St$. Similarly, we invoke $\langle s, x|\,\mathrm{e}^{-\mathrm{i}\epsilon SPt/\hbar} = \langle s, x - \epsilon st|$, the cyclicity of the trace and the product initial state (3) to get the matrix elements of $\rho_{\mathcal{PS}}(t)$ in the joint eigenbasis $\{|s, x\rangle\}$ of S and X,

$$\langle s, x|\rho_{\mathcal{PS}}(t)|s', x'\rangle \simeq \langle s|\rho_{\mathcal{S}}^0(t)|s'\rangle\langle x|\rho_{\mathcal{P}}^{ss'}(t)|x'\rangle, \qquad t \ll T_{\mathcal{S}}, T_{\mathcal{P}}, \tag{9}$$

with

$$\rho_{\mathcal{S}}^0(t) = \mathrm{e}^{-\mathrm{i}H_{\mathcal{S}}t/\hbar}\rho_{\mathcal{S}}(0)\,\mathrm{e}^{\mathrm{i}H_{\mathcal{S}}t/\hbar} \tag{10}$$

evolving as for the free object, while the pointer matrix elements

$$\langle x|\rho_{\mathcal{P}}^{ss'}(t)|x'\rangle = \langle x_s(t)|\,\mathrm{tr}_{\mathcal{B}}\,\tilde{U}_{sx}(t)\rho_{\mathcal{PB}}(0)\tilde{U}_{s'x'}(t)^\dagger|x'_{s'}(t)\rangle \tag{11}$$

involve the bath evolution operator and shifted positions

$$\tilde{U}_{sx}(t) = \left(\mathrm{e}^{-\mathrm{i}\int_0^t \mathrm{d}\tau x_s(t-\tau)\tilde{B}(\tau)/\hbar}\right)_+, \qquad x_s(t) = x - \epsilon st, \qquad x'_{s'}(t) = x' - \epsilon s't. \tag{12}$$

Note that $\mathrm{e}^{-\mathrm{i}H_{\mathcal{P}}t/\hbar}\rho_{\mathcal{PB}}(0)\,\mathrm{e}^{\mathrm{i}H_{\mathcal{P}}t/\hbar} \simeq \rho_{\mathcal{PB}}(0)$ for $t \ll T_{\mathcal{P}}$. The Hamiltonian $H_{\mathcal{S}}$ cannot be neglected in (10), even for $t \ll T_{\mathcal{S}}$, although for such times $\langle s|\rho_{\mathcal{S}}^0(t)|s\rangle \simeq \langle s|\rho_{\mathcal{S}}|s\rangle$.

To evaluate the matrix elements (11) we use the high-temperature approximation

$$\rho_{\mathcal{PB}}(0) \simeq Z_{\mathcal{PB}}^{-1}\,\mathrm{e}^{-\beta H_{\mathcal{P}}/2}\,\mathrm{e}^{-\beta(H_{\mathcal{B}}+H_{\mathcal{PB}})}\,\mathrm{e}^{-\beta H_{\mathcal{P}}/2} \tag{13}$$

for the pointer–bath Gibbs state (3). Given the weak-coupling condition $\Delta_{\mathrm{th}}^2\beta^2\langle B^2\rangle_0 < 1/2$ implying the stability (7) (thanks to $\gamma_0 \leqslant \hbar\beta\langle B^2\rangle_0/2$, see [18]), the error incurred is $\mathrm{O}\big(\hbar^2\beta^2/T_{\mathcal{P}}^2\big)$, as easily seen from the Baker–Campbell–Haussdorf formula. Thus

$$\langle x|\rho_{\mathcal{P}}^{ss'}(t)|x'\rangle \propto \int \mathrm{d}y\langle x_s(t)|\,\mathrm{e}^{-\beta H_{\mathcal{P}}/2}|y\rangle\langle y|\,\mathrm{e}^{-\beta H_{\mathcal{P}}/2}|x'_{s'}(t)\rangle Z_{\mathcal{B},y}\langle\tilde{U}_{s'x'}(t)^\dagger\tilde{U}_{sx}(t)\rangle_y \tag{14}$$

where $\langle\cdot\rangle_y$ is the bath average with respect to the modified equilibrium state $\rho_{\mathcal{B},y} = Z_{\mathcal{B},y}^{-1}\,\mathrm{e}^{-\beta(H_{\mathcal{B}}+yB)}$; the normalization factor is determined by using the QCLT as [18]

$$Z_{\mathcal{B},y} = \mathrm{e}^{\beta\gamma_0 y^2/\hbar}Z_{\mathcal{B},0}. \tag{15}$$

At this point we momentarily pause with dynamics and show that at $t = 0$, when $\tilde{U}_{s'x'} = \tilde{U}_{sx} = 1$, $x_s = x$ and $x'_{s'} = x'$, (14) yields the initial pointer state announced in (4). To that end we invoke high temperatures $\hbar\beta \ll T_{\mathcal{P}}$ again to approximate the matrix element $\langle x|\mathrm{e}^{-\beta H_{\mathcal{P}}/2}|y\rangle$ by $\mathrm{e}^{-\beta(V(x)+V(y))/4}\mathrm{e}^{-4\pi^2(x-y)^2/\lambda_{\mathrm{th}}^2}$. Replacing $V(y)$ by $V''(0)y^2/2$ in that expression and doing the Gaussian y-integral in (14), we arrive at the initial state (4) by neglecting terms $\mathrm{O}\big(\lambda_{\mathrm{th}}^2/\Delta_{\mathrm{th}}^2, \lambda_{\mathrm{th}}^2/\Delta_{\mathrm{eff}}^2\big)$.

Let us return to the time-evolved pointer matrix (14). Since $\rho_{\mathcal{B},y}$ factors into single-degree-of-freedom states, the QCLT assigns Gaussian statistics to the bath coupling agent B for the average $\langle\cdot\rangle_y$, with a mean $\langle\tilde{B}(\tau)\rangle_y \propto y$ given by linear response theory and a variance *independent of* y [18],

$$\langle\tilde{B}(t)\rangle_y = -\frac{2y}{\hbar}\int_{-\infty}^t \mathrm{d}\tau\Im h(\tau), \qquad \langle\tilde{B}(t)\tilde{B}(t')\rangle_y - \langle\tilde{B}(t)\rangle_y\langle\tilde{B}(t')\rangle_y = h(t-t'). \tag{16}$$

Therefore, $\langle\tilde{U}_{s'x'}(t)^\dagger\tilde{U}_{sx}(t)\rangle_y$ coincides with its value for $y = 0$ up to a phase factor $\mathrm{e}^{\mathrm{i}\int_0^t \mathrm{d}\tau\langle\tilde{B}(t-\tau)\rangle_y(x'_{s'}(\tau)-x_s(\tau))/\hbar}$ and the y-integral in (14) remains Gaussian for $t > 0$.

Of prime importance is the *decoherence factor*

$$\langle\tilde{U}_{s'x'}(t)^\dagger\tilde{U}_{sx}(t)\rangle_0 = \mathrm{e}^{-D_t(x_s(t),x'_{s'}(t);s,s')-\mathrm{i}\phi_t(x,x';s,s')} \tag{17}$$

with a *positive decoherence exponent* D_t revealed by the QCLT as [17, 18]

$$D_t(x, x'; s, s') = \frac{1}{\hbar^2}\int_0^t \mathrm{d}\tau_1 \int_0^{\tau_1} \mathrm{d}\tau_2\Re h(\tau_1 - \tau_2) \times (x'_{s'}(-\tau_1) - x_s(-\tau_1))(x'_{s'}(-\tau_2) - x_s(-\tau_2)) \tag{18}$$

and a real phase ϕ_t irrelevant for decoherence. Let us stress that the aforementioned results (in particular (18)) are exact (not lowest order in the pointer–bath coupling). They are consequences of Wick's theorem as implied by the QCLT and the additivity (2) of the bath coupling agent B. Direct proofs of (15), (16) and (18) are easy in the particular case of a bath composed of harmonic oscillators linearly coupled to $\mathcal{P}$.

It turns out [18] that the phase factor $\langle \tilde{U}_{s'x'}(t)^\dagger \tilde{U}_{sx}(t)\rangle_y / \langle \tilde{U}_{s'x'}(t)^\dagger \tilde{U}_{sx}(t)\rangle_0$ entails nothing but a correction of relative order $(\lambda_{\rm th}/\Delta_{\rm eff})^2$ to the decoherence exponent D_t under the stability condition (7). Dropping that correction, the y-integral reduces to the initial pointer density matrix (4), albeit with the shifted pointer positions $x \to x_s(t) = x - \epsilon s t$ and $x' \to x'_{s'}(t) = x' - \epsilon s' t$ reflecting the action of the object–pointer coupling. Our final result for the object–pointer state at time $t \ll T_{\mathcal{S}}, T_{\mathcal{P}}$ is

$$\langle s, x|\rho_{\mathcal{PS}}(t)|s', x'\rangle = \langle s|\rho^0_{\mathcal{S}}(t)|s'\rangle \langle x_s(t)|\rho_{\mathcal{P}}(0)|x'_{s'}(t)\rangle \, \mathrm{e}^{-D_t(x_s(t),x'_{s'}(t);s,s')-\mathrm{i}\phi_t} \tag{19}$$

with the notations specified in (4), (10) and (18). Entanglement and decoherence contribute separately in that remarkably simple 'final state'; they lead respectively to the second and third factors in (19). The decoherence (third) factor equals unity for $s = s'$ and $x = x'$.

4. Discussion

Given the narrow peaks (of width $\Delta_{\rm eff}$) of the initial pointer density matrix (4) at $x = x' = 0$, one can appreciate the fate of the $s \neq s'$ coherences in the final state (19) by setting $x_s(t) = x'_{s'}(t) = 0$ there. The decoherence factor then reads

$$\mathrm{e}^{-D_t^{\rm peak}(s,s')} = \exp\left\{-\frac{\epsilon^2(s-s')^2}{\hbar^2}\int_0^t \mathrm{d}\tau_1 \int_0^{\tau_1} \mathrm{d}\tau_2 \tau_1 \tau_2 \Re h(\tau_1 - \tau_2)\right\} \tag{20}$$

and reveals irreversible decay as soon as the time t much exceeds the *decoherence time* $t_{\rm dec}(s, s')$; we may define that time implicitly as $D^{\rm peak}_{t_{\rm dec}}(s, s') = 1$. It can be shown [18] that $D_t^{\rm peak}(s, s')$ is an increasing convex function of time (see the inset in figure 1).

The diagonal ($s = s'$ and $x = x'$) terms in the final state (19) give the probability density of pointer positions for fixed s. That density has a sharp peak (of width $\Delta_{\rm eff}$) at $x = \epsilon s t$. The peaks associated with distinct s and s' begin to be resolved at the *entanglement time* $t_{\rm ent}(s, s') = \Delta_{\rm eff}(\epsilon|s - s'|)^{-1}$. That time is related to $t_{\rm dec}(s, s')$ by

$$\left(\frac{t_{\rm ent}(s,s')}{\eta}\right)^2 = \frac{1}{(\hbar\beta)^2}\int_0^{t_{\rm dec}(s,s')} \mathrm{d}\tau_1 \int_0^{\tau_1} \mathrm{d}\tau_2 \tau_1 \tau_2 \frac{\Re h(\tau_1 - \tau_2)}{\langle B^2\rangle_0} \tag{21}$$

where $\eta = \langle B^2\rangle_0^{1/2}\Delta_{\rm eff}\beta$ is a dimensionless measure of the strength of the pointer–bath coupling. Figure 1 shows $t_{\rm dec}$ as function of $t_{\rm ent}/\eta$ for three distinct choices of the bath correlator $\Re h(t)/\langle B^2\rangle_0$.

Let us note that, in analogy with the results of [14], the $s \neq s'$ matrix elements of the reduced density matrix of $\mathcal{S}$, $\langle s|\,\mathrm{tr}_{\mathcal{P}}\,\rho_{\mathcal{PS}}(t)|s'\rangle$, decay to zero on a time scale $\lambda_{\rm th}(\epsilon|s - s'|)^{-1}$ much shorter than both $t_{\rm ent}(s, s')$ and $t_{\rm dec}(s, s')$ (see (4) and (19)).

Recalling that S has a discrete spectrum, we denote by δs the minimum of $|s - s'|$ over all pairs (s, s') of eigenvalues present in the object initial state (we suppose that $\langle s|\rho_{\mathcal{S}}|s'\rangle = 0$ if s and s' belong to a part of the spectrum containing arbitrarily close eigenvalues, near an accumulation point, so that $\delta s > 0$). At time $t > t_{\rm ent} = \Delta_{\rm eff}(\epsilon\delta s)^{-1}$, neighbouring peaks of the pointer densities can be resolved. Each eigenvalue s of the measured object observable is then uniquely tied up with 'its' pointer position $\epsilon s t$. If t is also much larger than the

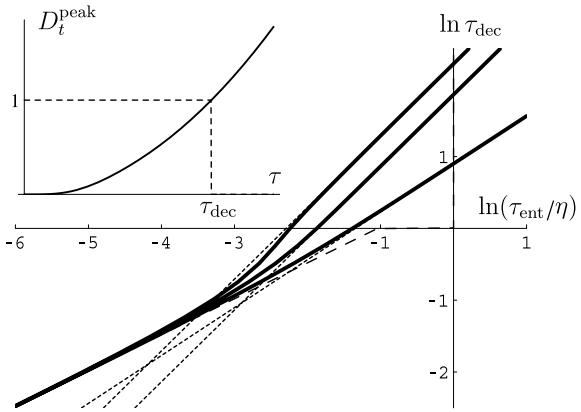

Figure 1. Decoherence time $\tau_{\rm dec} = t_{\rm dec}/T_{\mathcal{B}}$ in units of $T_{\mathcal{B}}$ against $t_{\rm ent}/(\eta T_{\mathcal{B}})$ in a log–log scale. We take $\widehat{\Re h}(\omega) = \mathrm{i}\coth(\hbar\beta\omega/2)\widehat{\Im h}(\omega)$ (KMS relation) and $\widehat{\Im h}(\omega) \propto \omega^m\, \mathrm{e}^{-(\hbar\beta\omega/5)^2}$. The larger decay time of the bath correlator $h(t)$ is then the thermal time $T_{\mathcal{B}} = \hbar\beta$. Solid curves: $m = 5, 3, 1$ (from left to right). Broken curves: approximate expressions (22)–(23) for $\tau_{\rm dec} \ll 1$ (dashed lines) and $\tau_{\rm dec} \gg 1$ (dotted lines). Inset: decoherence exponent $D_t^{\rm peak}$ against $\tau = t/T_{\mathcal{B}}$ ($m = 3$).

maximum decoherence time $t_{\rm dec} = t_{\rm dec}(s, s \pm \delta s)$ (t being still smaller than $T_{\mathcal{S}}$ and $T_{\mathcal{P}}$), the matrix elements (19) for $s \neq s'$ almost vanish for all values of (x, x'). Assuming moreover that the spectrum of S is non-degenerate, object and pointer are in a separable mixed state, $\rho_{\mathcal{PS}}(t) \simeq \sum_s p_s |s\rangle\langle s| \otimes \rho_{\mathcal{P}}^{ss}(t)$, with $p_s = \langle s|\rho_{\mathcal{S}}^0(t)|s\rangle \simeq \langle s|\rho_{\mathcal{S}}|s\rangle$. Hence, according to the Born rule, with probability p_s the object is in the eigenstate $|s\rangle$ and the pointer is in a state $\rho_{\mathcal{P}}^{ss}(t)$ localized near $x = \epsilon s t$ with probability density $\langle x|\rho_{\mathcal{P}}^{ss}(t)|x\rangle \propto \mathrm{e}^{-\beta V_{\rm eff}(x-\epsilon s t)}$, in agreement with the von Neumann postulate. The coupling $H_{\mathcal{PS}}$ may be switched off at time $t_{\rm int} \approx W_{\rm eff}(\epsilon\delta s)^{-1} \gg t_{\rm ent}$, where $W_{\rm eff}$ is defined in subsection 2.2. Then all pointer states $\rho_{\mathcal{P}}^{ss}(t)$ are outside the effective potential well saved for one eigenvalue $s = 0$. The inter-peak distance is amplified at time $t > t_{\rm int}$ by the effective pointer dynamics, till it reaches a macroscopically resolvable magnitude $\Delta_{\rm class}$. Then a pointer reading, while still a physical process in principle perturbing $\mathcal{P}$, surely cannot blur the distinction of the peaks.

5. Limiting regimes

Formula (21) explicitly yields the decoherence time in two interesting limits. An *interaction dominated* regime has decoherence outrunning bath correlation decay ($t_{\rm dec} \ll T_{\mathcal{B}}$) such that we can use $h(\tau) \simeq \langle B^2\rangle_0$ in (20) and (21). We conclude

$$\mathrm{e}^{-D_t^{\rm peak}(s,s')} = \mathrm{e}^{-(t/t_{\rm dec}(s,s'))^4}, \qquad \frac{t_{\rm dec}(s,s')}{\hbar\beta} = 2^{3/4}\left(\frac{t_{\rm ent}(s,s')}{\hbar\beta\eta}\right)^{1/2} \tag{22}$$

for $t_{\rm dec} \ll T_{\mathcal{B}}, T_{\mathcal{S}}, T_{\mathcal{P}}$. The decoherence time depends on the bath through the pointer–bath coupling strength η only. It is smaller than $t_{\rm int}$ when $t_{\rm ent} \geqslant 8^{1/2}\,\eta^{-1}(\Delta_{\rm eff}/W_{\rm eff})^2\hbar\beta$. The small-time behaviour (22) of the decoherence factor appears also in other models [14].

The opposite limit $t_{\rm dec} \gg T_{\mathcal{B}}$ defines the *Markovian regime*. A rotating-wave approximation is inappropriate due to our restriction (1). Assuming that the Fourier transform of the imaginary part of h behaves as $(\widehat{\Im h})(\omega) \propto \omega^m$ when $\omega \to 0$, we find

$$\mathrm{e}^{-D_t^{\rm peak}(s,s')} = \mathrm{e}^{-(t/t_{\rm dec}(s,s'))^\gamma}, \qquad \frac{t_{\rm dec}(s,s')}{\hbar\beta} = c_m^{1/\gamma}\left(\frac{t_{\rm ent}(s,s')}{\hbar\beta\eta}\right)^{2/\gamma} \tag{23}$$

for $T_{\mathcal{B}} \ll t_{\rm dec} \ll T_{\mathcal{S}}, T_{\mathcal{P}}$. Here $\gamma = 3$ for a 'Ohmic bath' ($m = 1$) and $\gamma = 2$ for a 'super-Ohmic bath' ($m \geqslant 3$); the constant c_m is independent of the strengths of the couplings (2), $c_1 = 3\hbar\beta\langle B^2\rangle_0 / \int_0^\infty {\rm d}\tau \Re h(\tau)$ and $c_{m\geqslant 3} = 2\hbar^2\beta^2\langle B^2\rangle_0 / |\int_0^\infty {\rm d}\tau\tau\Re h(\tau)|$.

In all cases the coherences decay non-exponentially. We compare in figure 1 the asymptotic results (22)–(23) with numerical solutions of (21). One finds an excellent agreement except for a thin range around $t_{\rm dec} = T_{\mathcal{B}} = \hbar\beta$. If $\eta \lesssim 1$, the only regime with a decoherence faster than resolution of pointer peaks ($t_{\rm dec} \leqslant t_{\rm ent}$) is the Markov regime ($t_{\rm ent} \geqslant c_1\eta^{-2}\hbar\beta$) with $m = 1$ (Ohmic bath). In all asymptotic regimes $t_{\rm dec} \leqslant t_{\rm int}$ if $\eta \geqslant c_m^{1/2}\Delta_{\rm eff}/W_{\rm eff}$ and $t_{\rm ent} \geqslant \hbar\beta \max\{(8/c_m)^{1/2}, 1\}\Delta_{\rm eff}/W_{\rm eff}$. This means that mesoscopic superpositions decay to mixtures faster than entanglement can create them.

6. Conclusion and outlook

We have investigated a model for quantum detection in which the entanglement produced by the coupling of the measured object with the pointer is simultaneous with decoherence of distinct pointer readouts; the apparatus (pointer and bath) is taken initially in a local thermal metastable state, not correlated to the object. We have shown that the decoherence time $t_{\rm dec}$ presents a universal behaviour in the interaction-dominated regime $t_{\rm dec} \ll T_{\mathcal{B}}$, whereas it depends strongly on the small-frequency behaviour of the bath correlator in the Markov regime $t_{\rm dec} \gg T_{\mathcal{B}}$. For reasonably strong pointer–bath and weak object pointer couplings, $t_{\rm dec}$ is smaller than the time $t_{\rm int}$ needed by entanglement to produce mesoscopic superpositions, which do not appear at any time during the measurement.

Several generalizations of our results present no difficulties. First, nonlinear pointer–bath couplings, $H_{\mathcal{PB}} = BX^\alpha$ with $\alpha > 1$, make richer decoherence scenarios and produce smaller decoherence times saved for Ohmic baths in the Markovian regime [18]. Second, the QCLT also works for baths of interacting particles if the correlator $\langle B_\mu B_\nu\rangle_0$ decays more rapidly than $1/|\mu - \nu|$ (see [19] for a related version of the QCLT in this context). We shall publish these elaborations elsewhere.

Acknowledgments

We acknowledge support by the Deutsche Forschungsgemeinschaft (through the project SFB/TR 12) and the Agence Nationale de la Recherche (project ANR-05-JCJC-0107-01) and thank M Guta for pointing out [16] to us.

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